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Injection Flows in a Heterogeneous Porous Medium

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International Conference on Progress in Fluid Dynamics and Simulation
Celebrating the 60th Birthday Anniversary of Tony Wen-Hann Sheu

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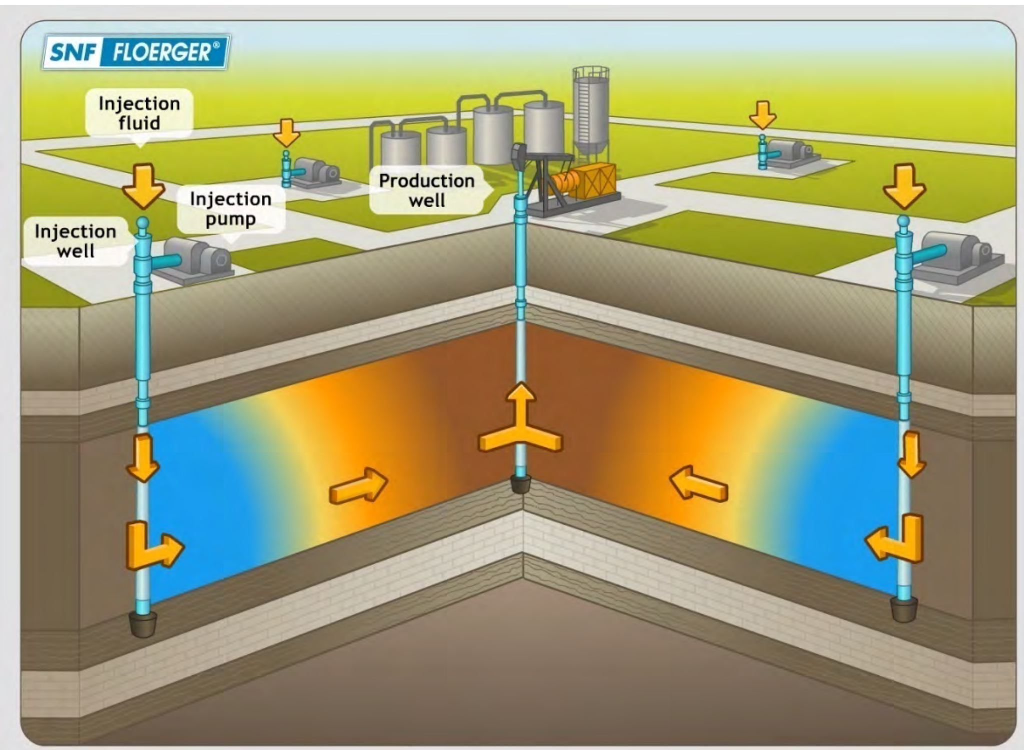
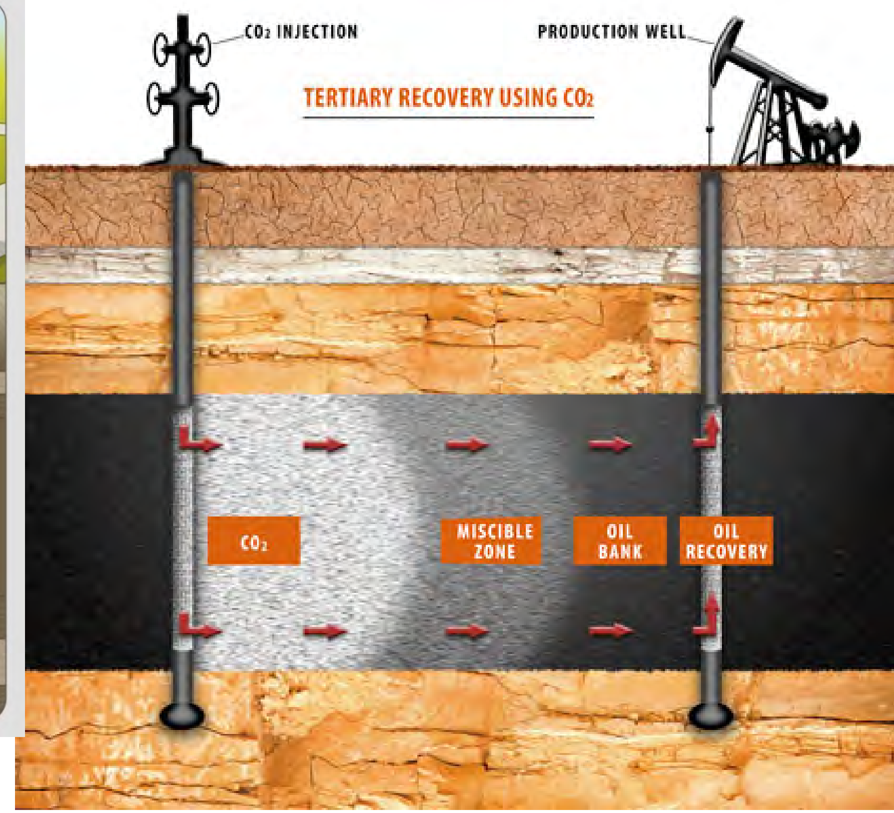
Prof. Tony Sheu,

Happy 60th Birthday Anniversary!

Viscous Fingering

- A less viscous fluid displacing a more viscous fluid
- Applications on Enhanced Oil Recovery & CO_2 Storage

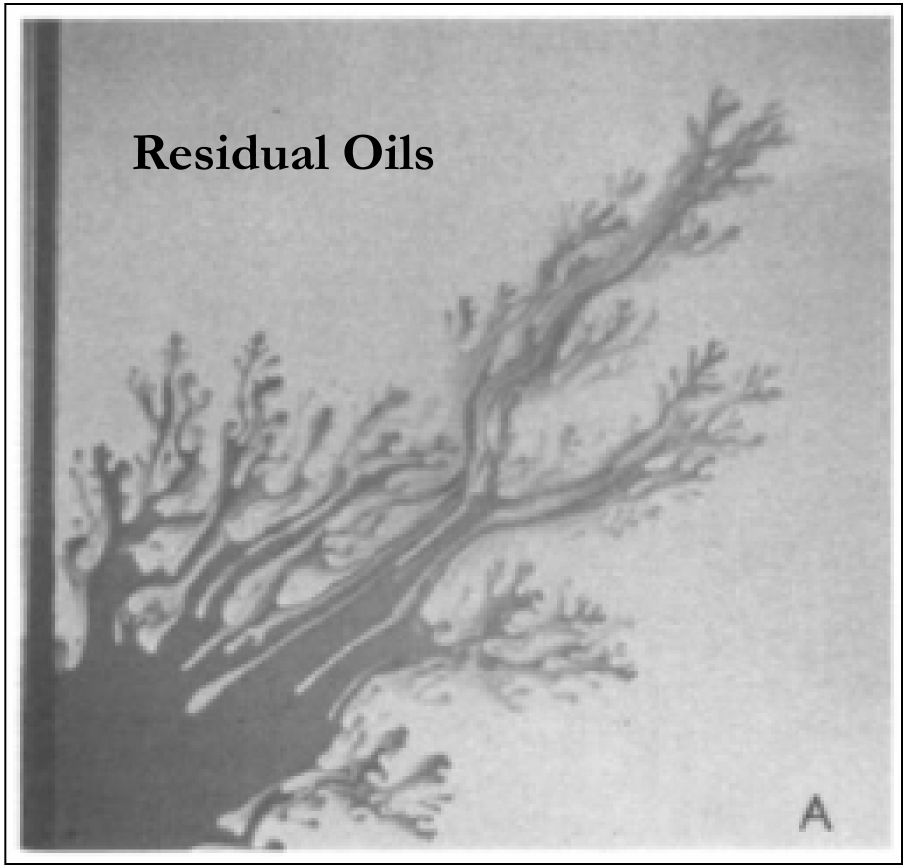




Sweeping Efficiency

Interfacial Instability (Viscous Fingering) leads to less efficient oil recovery, due to shield effects of the fingers. More residual oils are left behind.

Immiscible Rectilinear Displacement
(Maxworthy et al., 1986)



Miscible Five-Spot Displacement
(Claridge, 1986)

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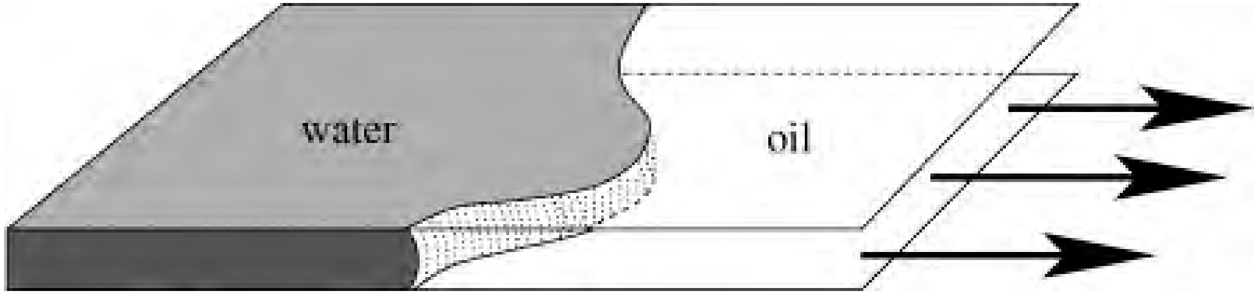
Hele-Shaw Cell and Porous Medium

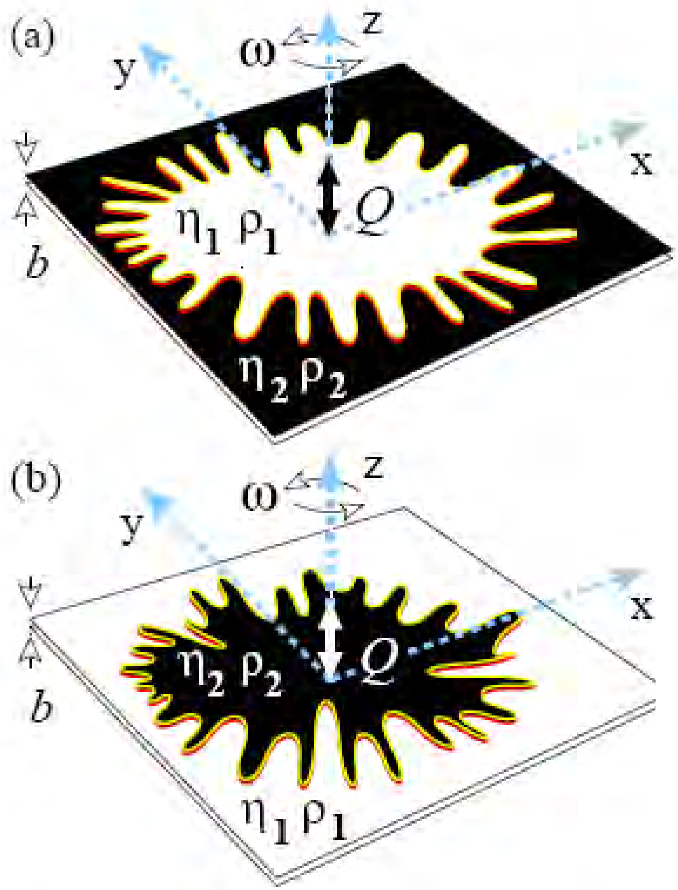


Hele-Shaw Equation $$\nabla P = -\frac{12\eta}{h^2}\mathbf{U}$$

Porous Medium : Darcy's Law $$\nabla P = -\frac{\eta}{k}\mathbf{U}$$

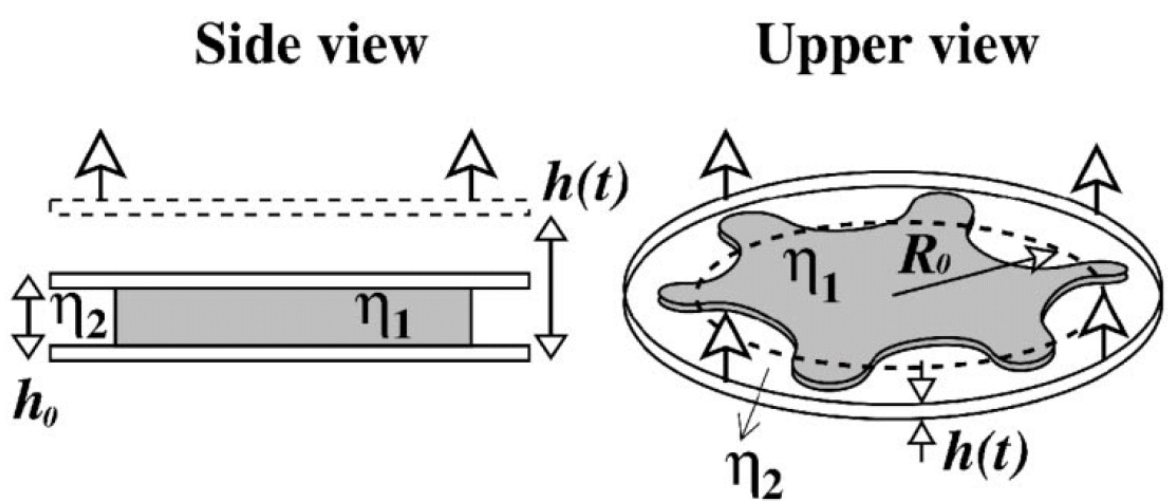
Hele-Shaw flow is mathematically similar to a 2-D porous medium flow ($k=h^2/12$).

Radial Hele-Shaw Flows



Radial flow configurations:

1. Rotation : density
2. Injection/Suction : viscosity
3. Vertical Lifting : viscosity



Phase Field Approach : Hele-Shaw-Cahn-Hilliard equations (associated with constant density)

$$\nabla \cdot \mathbf{u} = 0,$$

$$\nabla p = -\frac{12\eta}{h^2}\mathbf{u} - \epsilon\rho\nabla \cdot [(\nabla c)(\nabla c)^T],$$

$$\rho\left(\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c\right) = \alpha\nabla^2\mu,$$

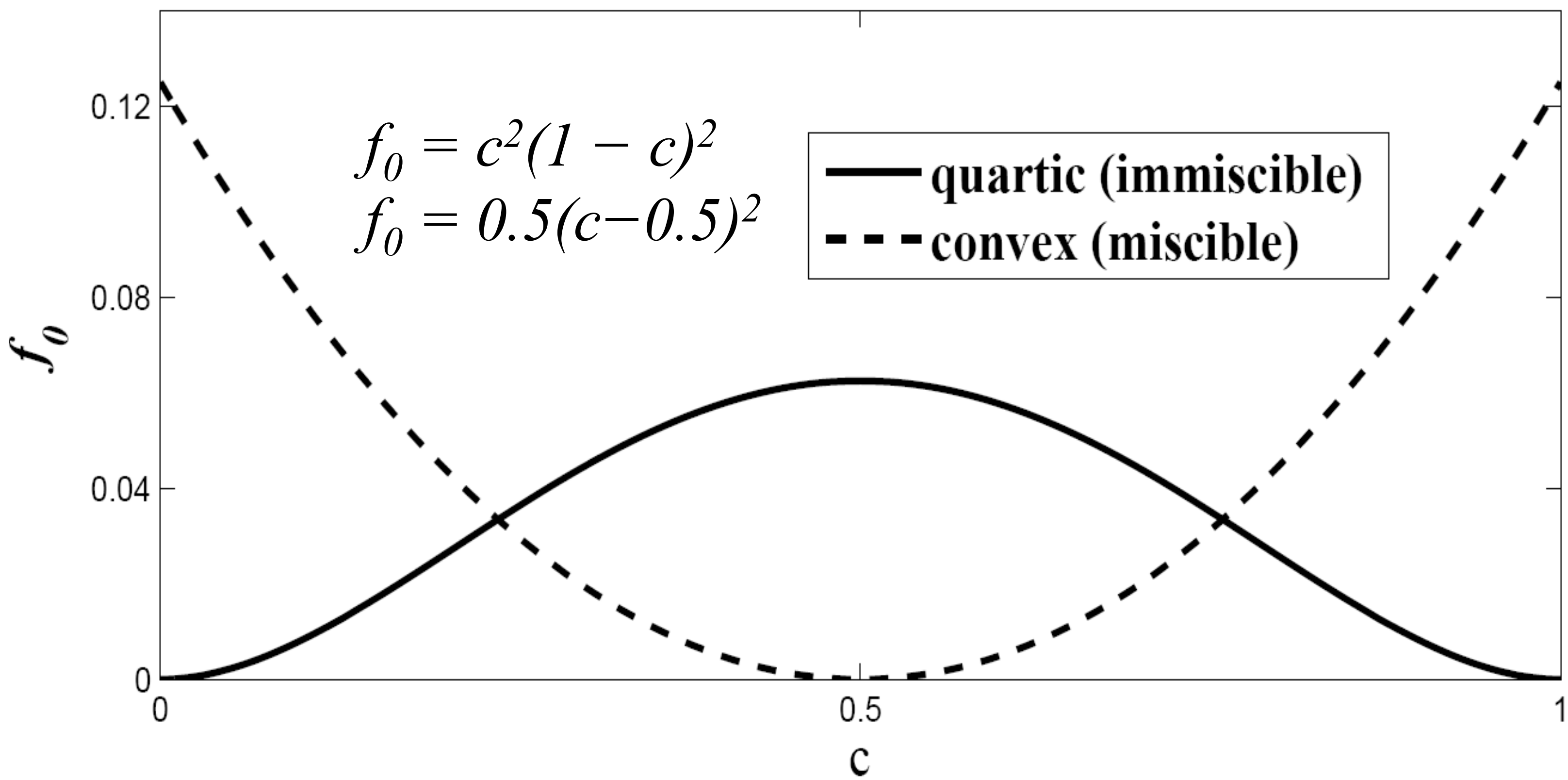
$$\mu = \frac{\partial f_0}{\partial c} - \epsilon\nabla^2 c.$$

Surface free energy

$$E = \rho\int\left(f_0 + \frac{\epsilon}{2}(\nabla c)^2\right)dV.$$

Miscibility

Profiles of specific interfacial free energy f_0



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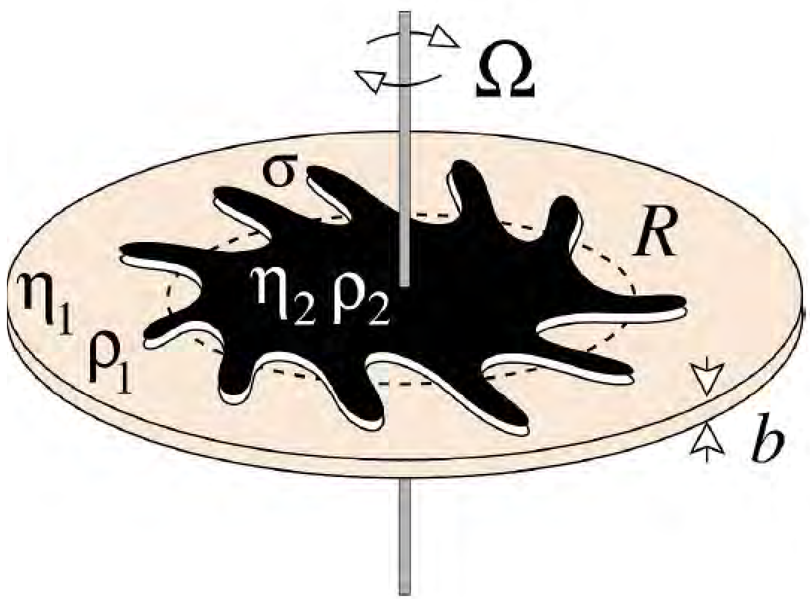
Immiscible Rotating Flows (Chen etal., PRE 2011)

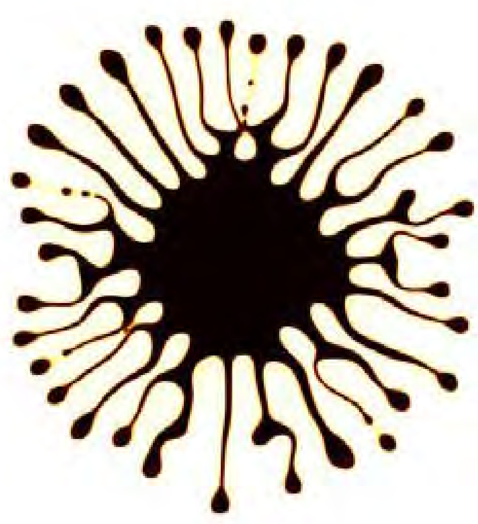


Immiscible Rotation

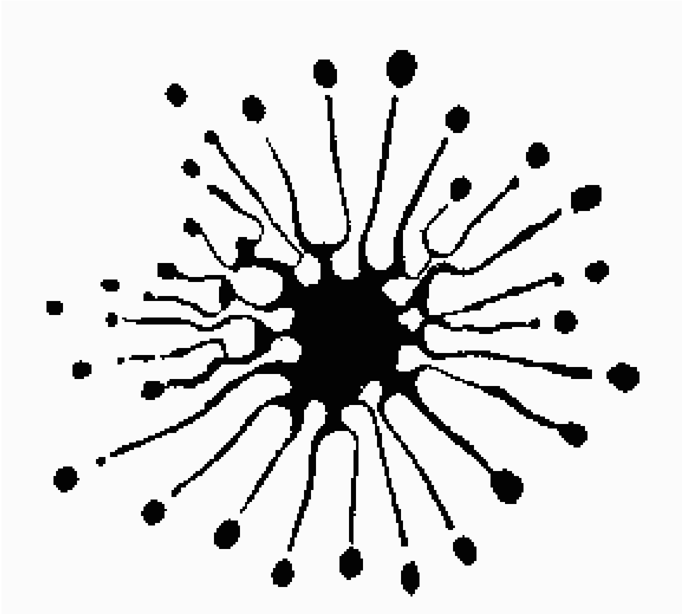
Specific free energy : $f_0 = c^2(1 - c)^2$

Surface tension : $\sigma = \frac{1}{\mathrm{Ga}} \int \left[f_0 + \frac{C}{2} \left(\frac{\partial c}{\partial \zeta} \right)^2 \right] d\zeta$

Rotating Bond number : $Bo_e = \frac{\sqrt{C/2}}{3\mathrm{Ga}}$.

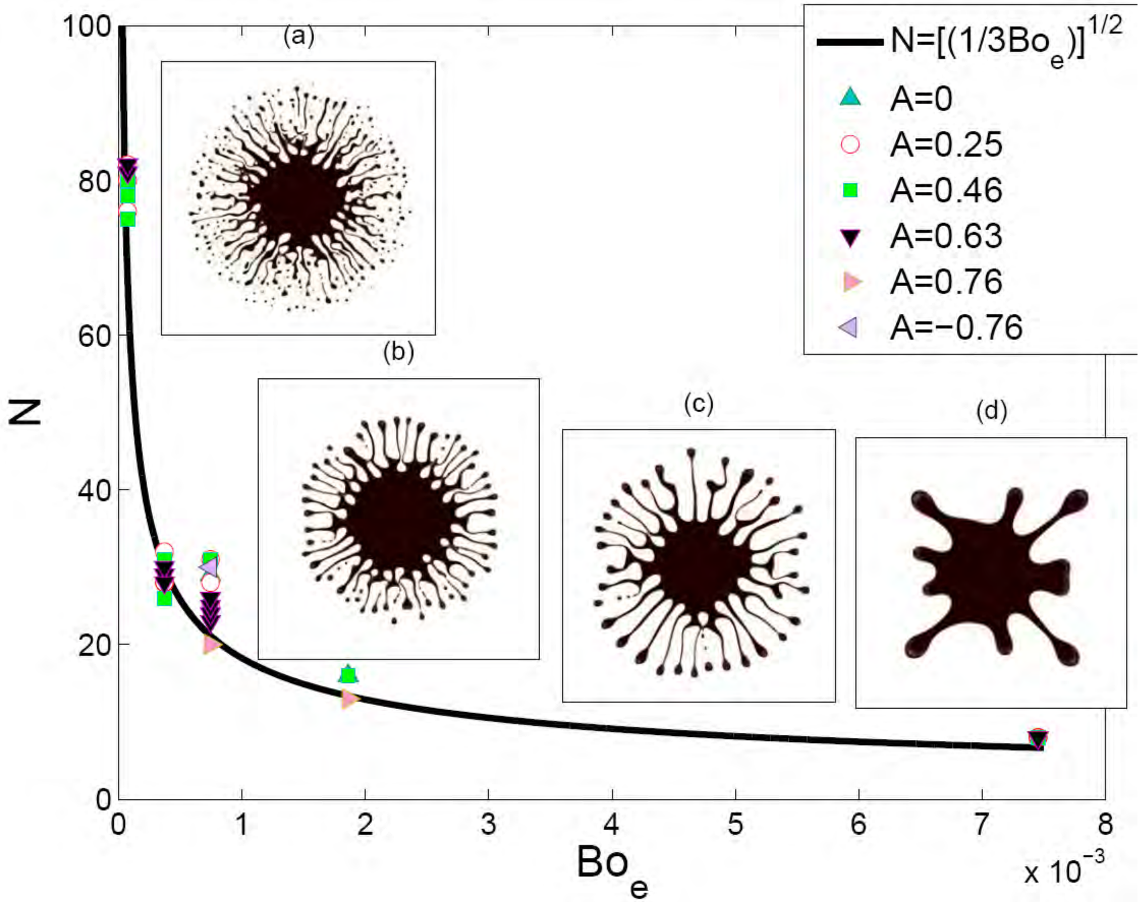


Present simulation : A=0.46, Bo=7.45x10^{-4}



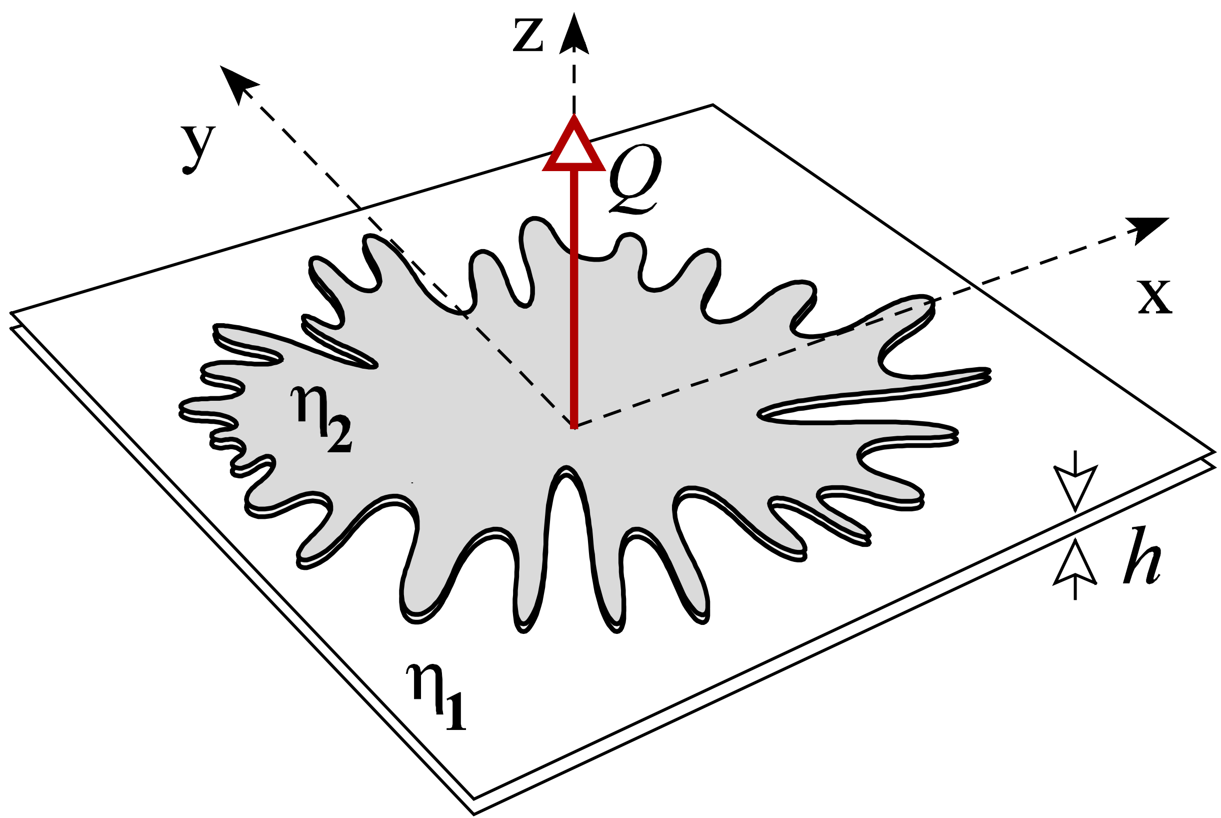
Experiment : A=0.4~0.5, Bo~4x10^{-4}
(Alvarez-Lacalle et al., 2004)

Number of Fingers vs. Bo

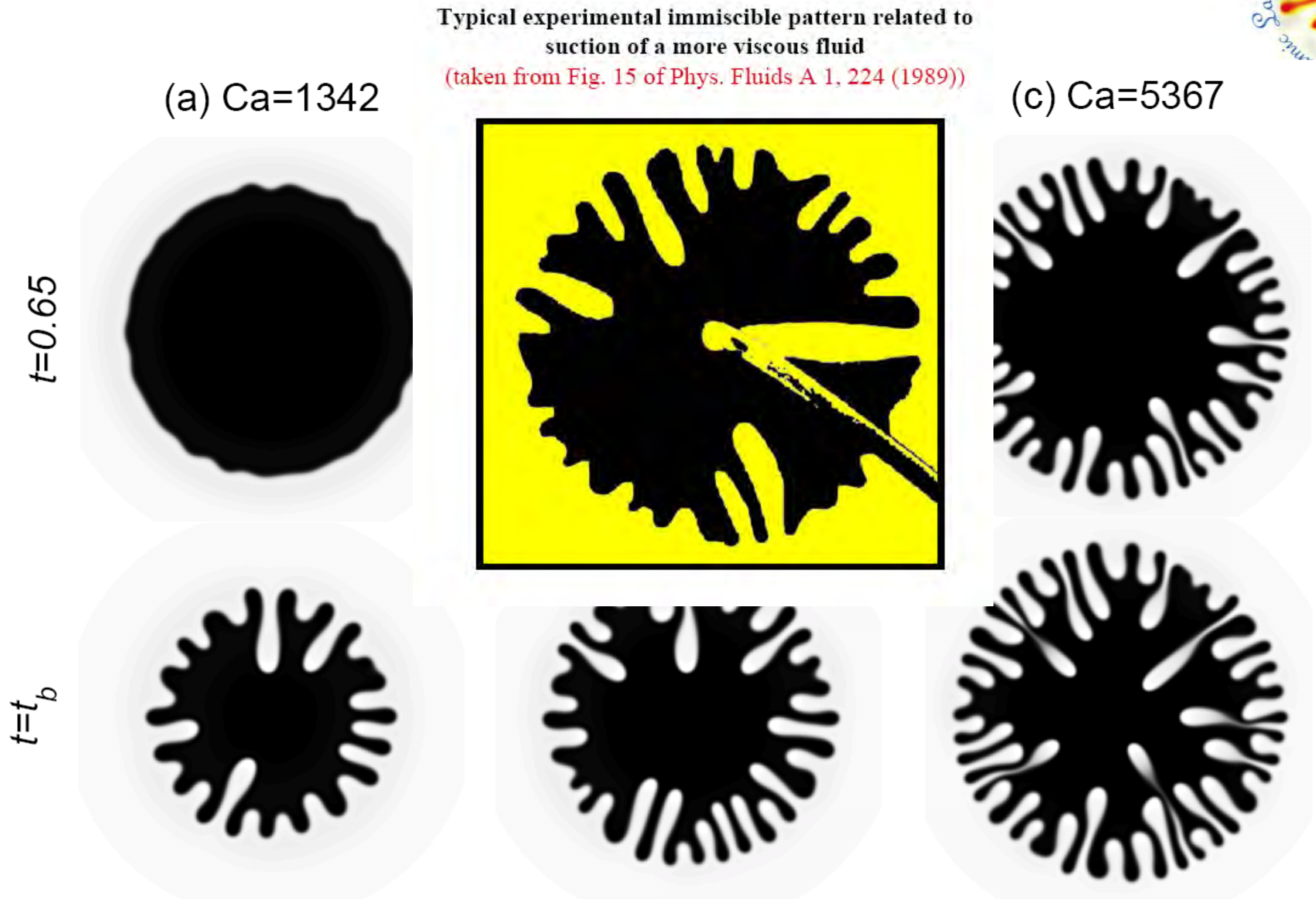


(1) Smaller values of Bo lead to a larger number of fingering structures.
(2) Favored occurrence of droplet emissions when Bo is decreased.
(3) Excellent agreement with the analytical results is achieved.

Immiscible Suction Flows (Chen et al., PRE 2014)

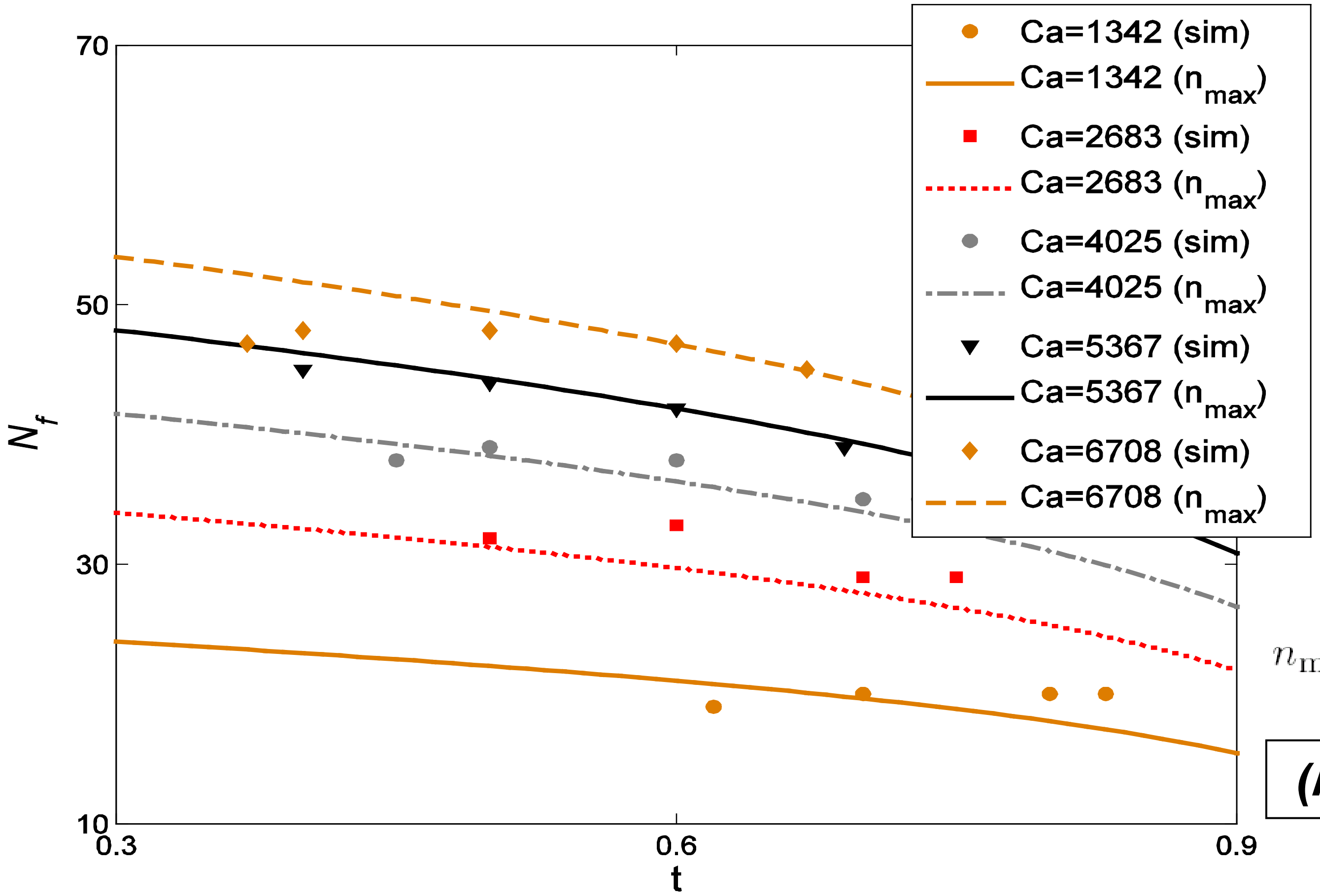


Immiscible Suction



Numerical simulations showing typical fingering patterns at times t=0.65, and the breakthrough time t_b, for increasingly larger values of the capillary number: Ca=1342 (t_b =0.834), Ca=2683 (t_b=0.752), and Ca=536 (t_b=0.684).

Number of Fingers vs. Ca



$$n_{\max}(r) \approx \sqrt{\frac{1}{3}\left(1 + \frac{e^R \mathrm{Ca}}{8} r\right)}$$

(Miranda & Widom, 1998)

Time evolution of the number of fingers N_f for different values of the capillary number Ca. Corresponding analytical predictions for the number of fingers (n_{max}) are also shown. Good general agreement is obtained.

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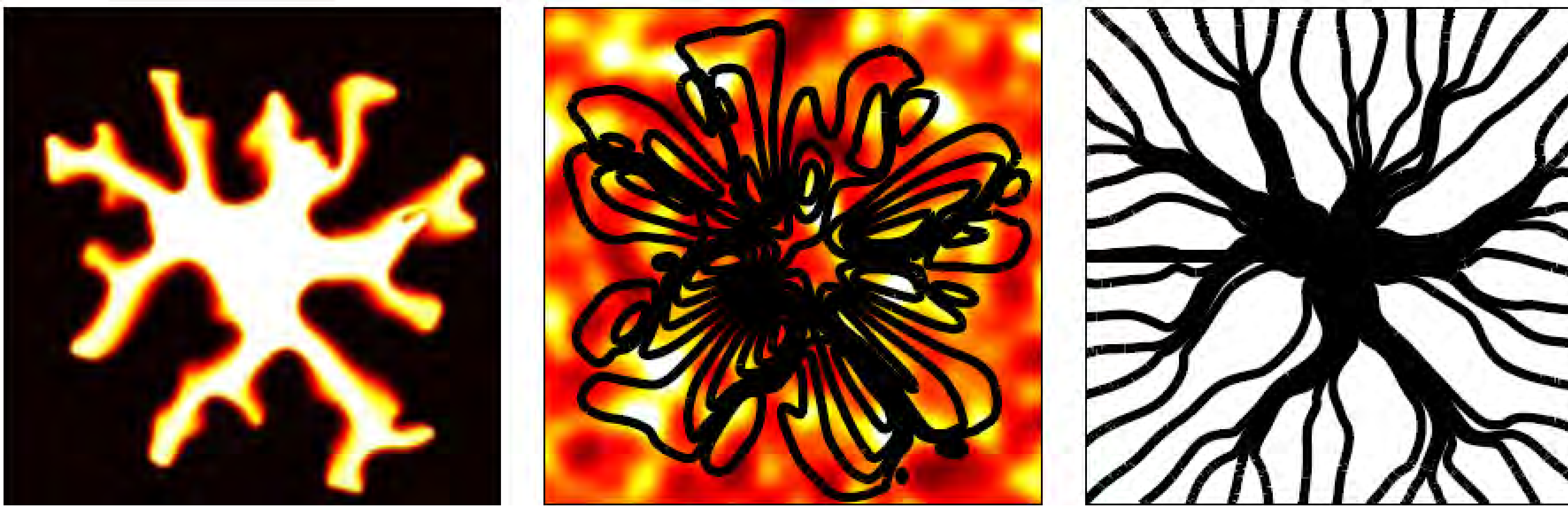
Miscible/Immiscible Injections in Heterogeneous Porous Medium



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Governing Equations

$$\nabla \cdot \mathbf{u} = 0,$$

$$\nabla p = -\frac{\eta}{k}\mathbf{u} - \frac{C}{I}\nabla \cdot \left[(\nabla c)(\nabla c)^T\right],$$

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = \frac{1}{\mathrm{Pe}}\nabla^2 \mu,$$

$$\mu = \frac{\partial f_0}{\partial c} - C\nabla^2 c.$$

Permeability (Shinozuka & Jen 1972, Chen & Meiburg 1998)

$$k(x,y) = Ke^g, \ \ g(x,y) = s^2 exp\left(-\pi[(\frac{x}{l})^2 + (\frac{y}{l})^2]\right)$$

$$\mathrm{Pe} = \frac{\rho D_f^2}{\alpha f^* t_f}, \ A = \frac{e^R - 1}{e^R + 1}, \ C = \frac{\epsilon}{D_f^2 f^*}, \ I = \frac{\eta_1 D_f^2}{\rho f^* K t_f}.$$

Dimensionless control parameters $Ca = \frac{3I}{\sqrt{C/2}}.$

Numerical Schemes

- **Phase Concentration equation :**
  - 3^{rd} order Runge-Kutta procedure in time
  - 6^{th} order compact finite difference in space

- **Hele-Shaw equations : Vorticity-Streamfunction**

- **potential velocity** $\mathbf{u}_{\text{pot}} = -\frac{Q}{2\pi r}[1 - \exp(-4r^2/{D_c}^2)]\hat{\mathbf{r}},$

- **vorticity equation** : 6^{th} order compact finite difference

- **Poisson (streamfunction) equation** :

  y-direction - 6^{th} order compact finite difference

  x-direction pseudo-spectral scheme

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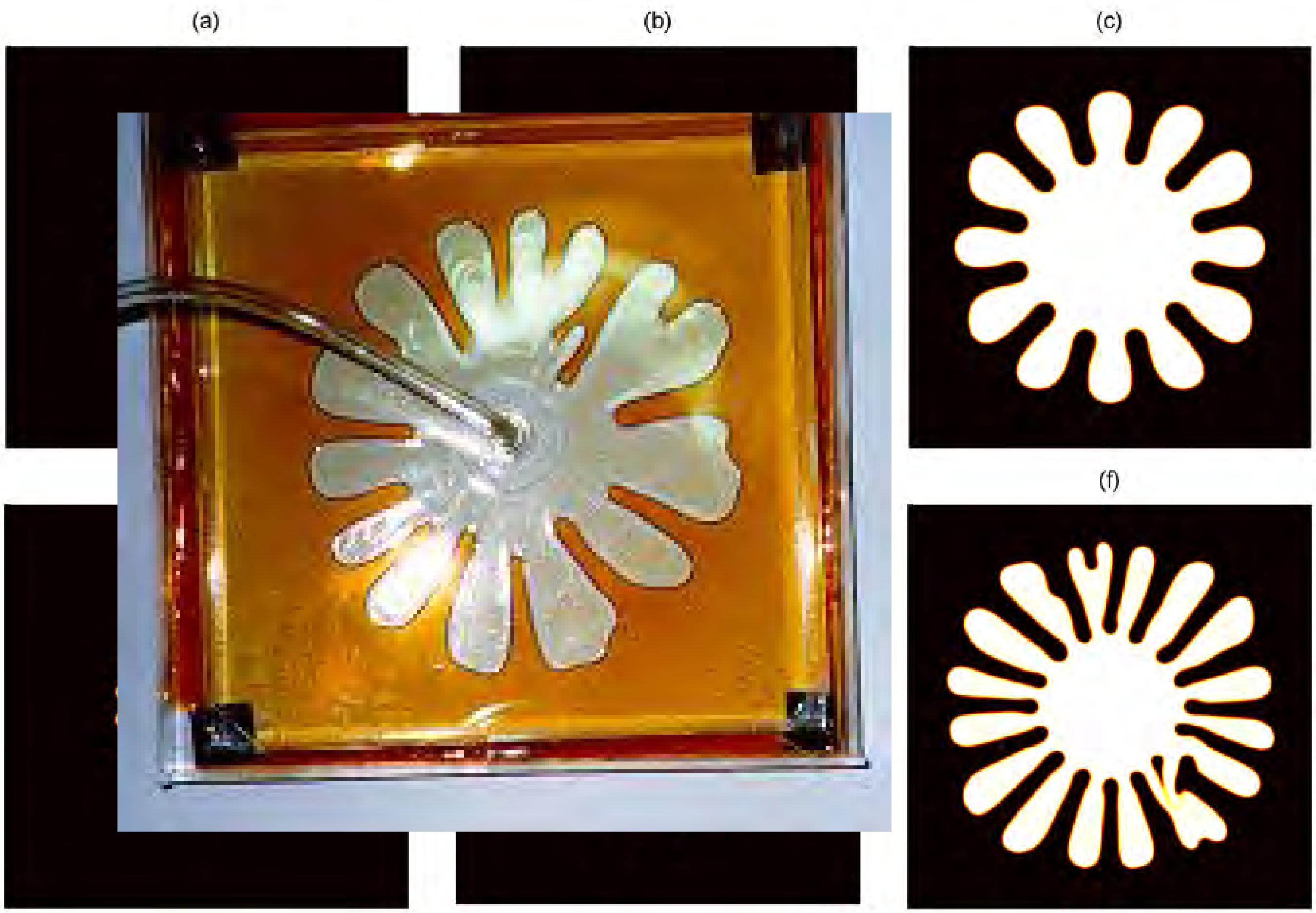
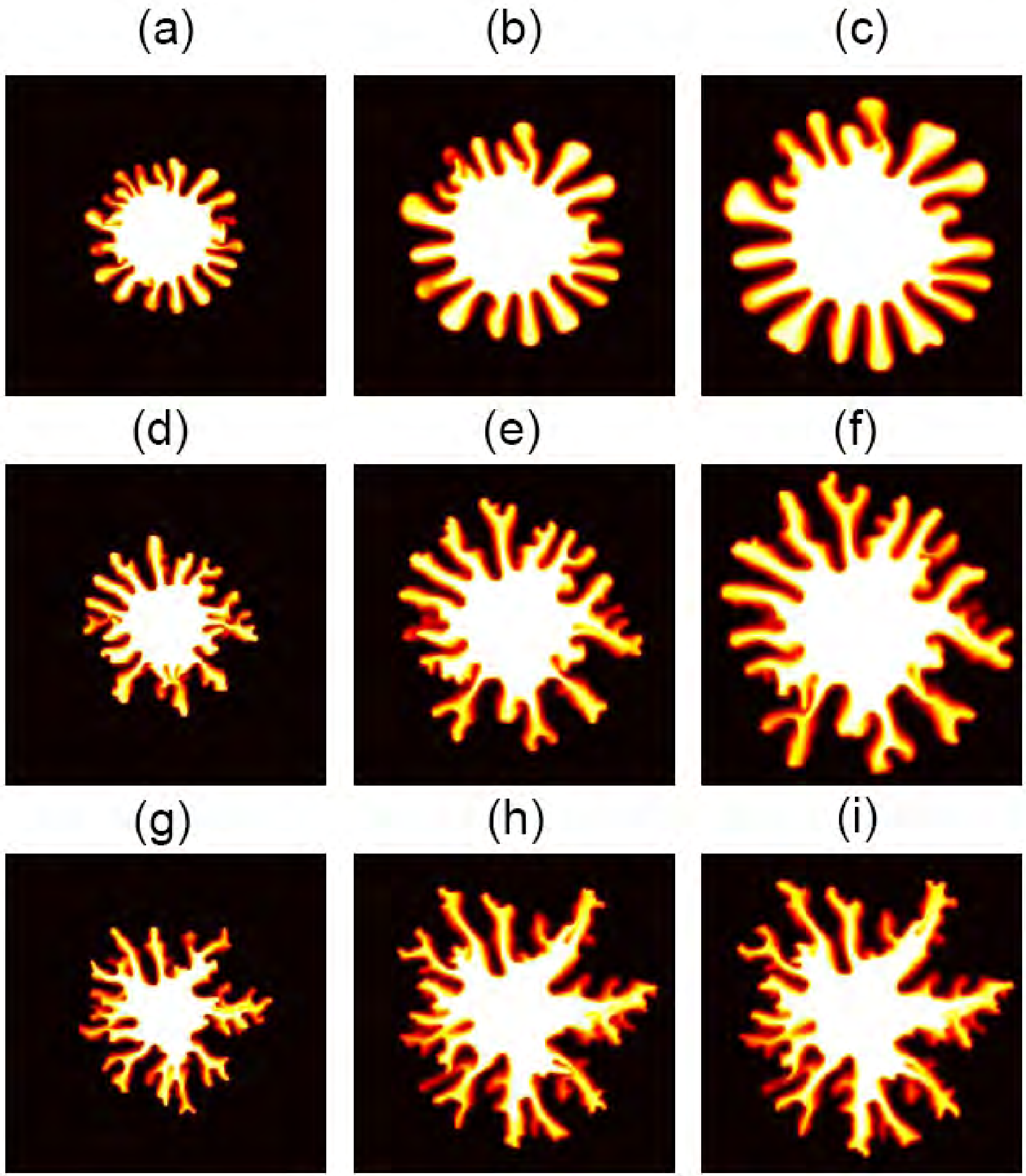
Immiscible/Homogeneous (s=0) - Surface Tension



Pe=3E3, A=0.922, C=1E-5, D_0=0.15, I=1&5

Miscible Injection : *l=0.02*

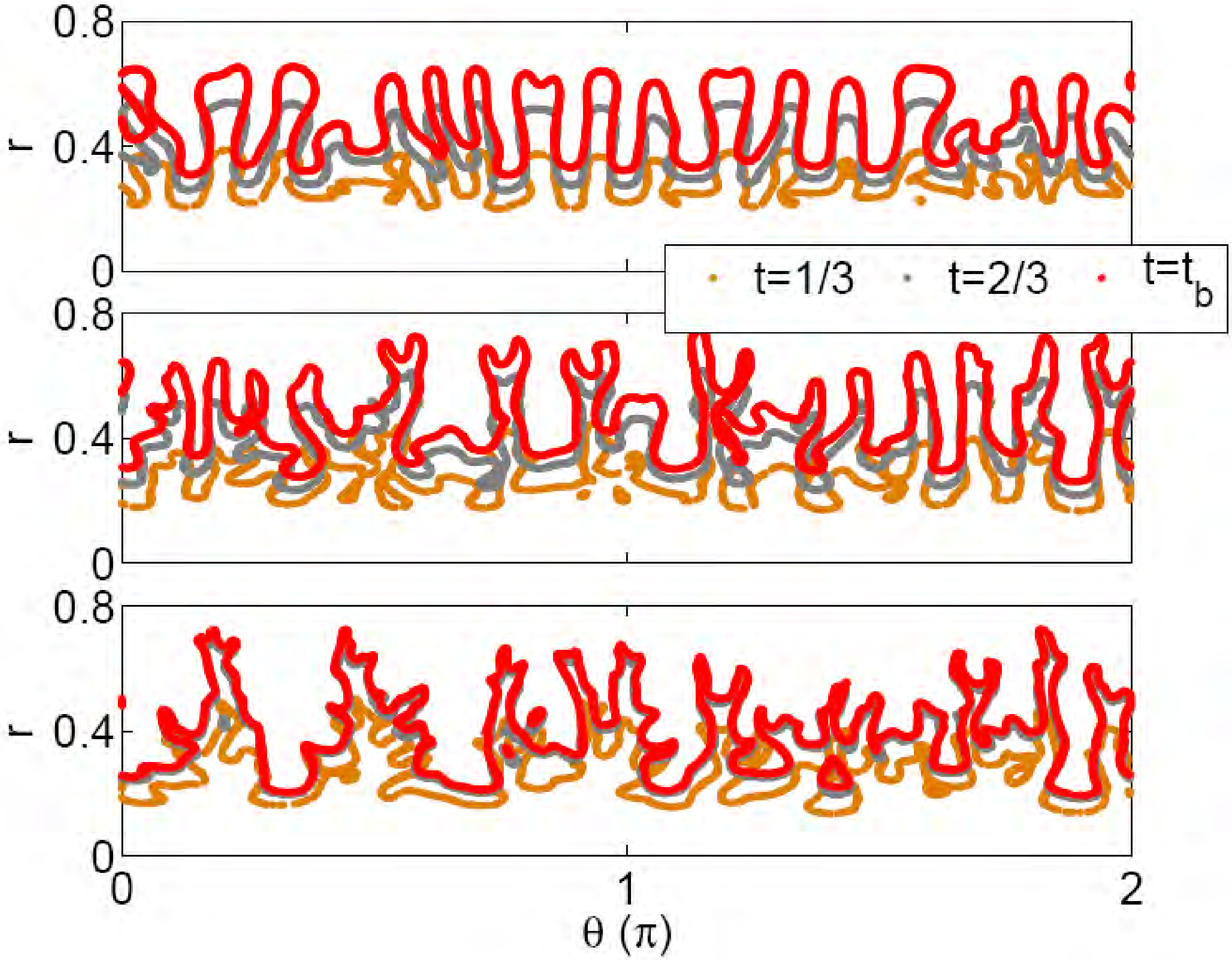


s=0 (Homo)

s=0.3

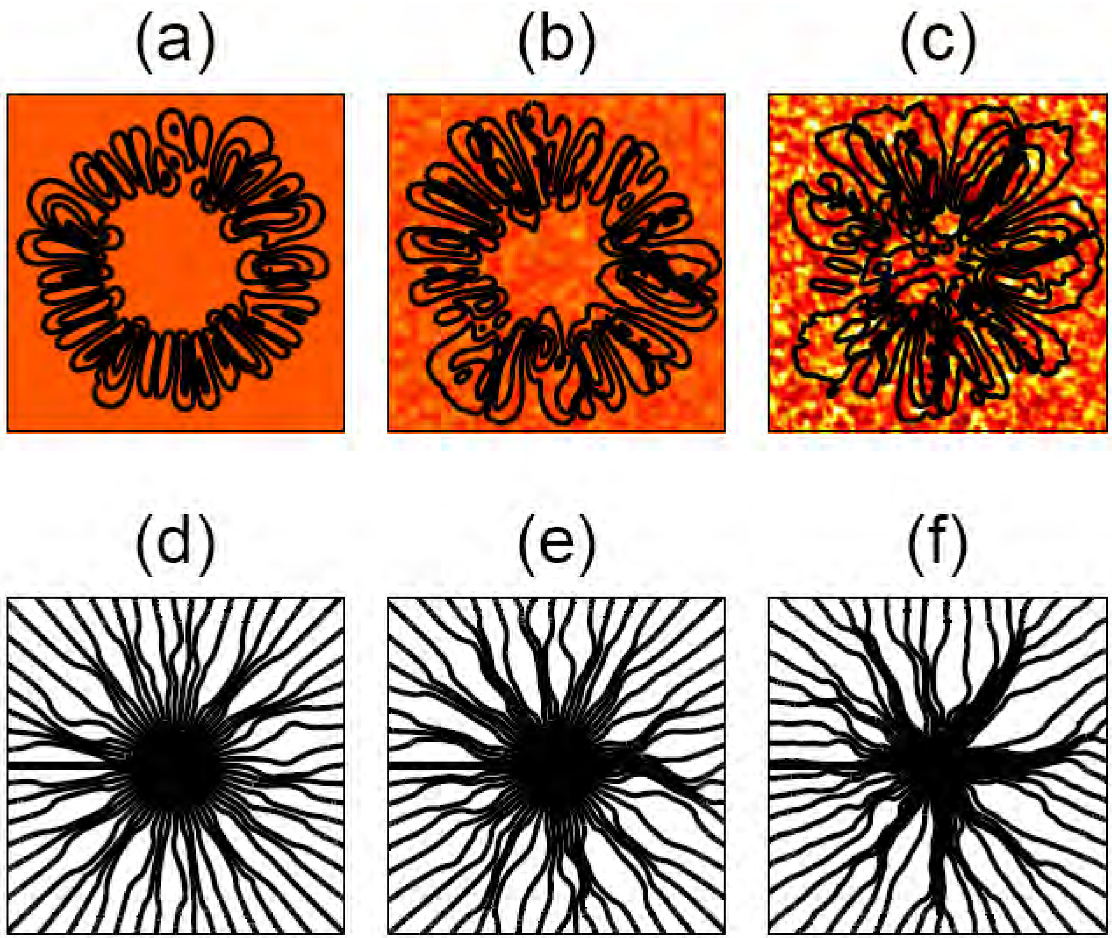
s=0.6

Miscible Injection : *l*=0.02



Evolutions of fingering interfaces presented in a polar coordinate at t=1/3, 2/3 and terminated time for s=0, 0.3 and 0.6.

Miscible Injection : *l=0.02*



The rotational component of streamlines superimposed on the correspondent permeability distribution (top row) and total streamlines (bottom row) for s=0, s=0.3 and s=0.6.

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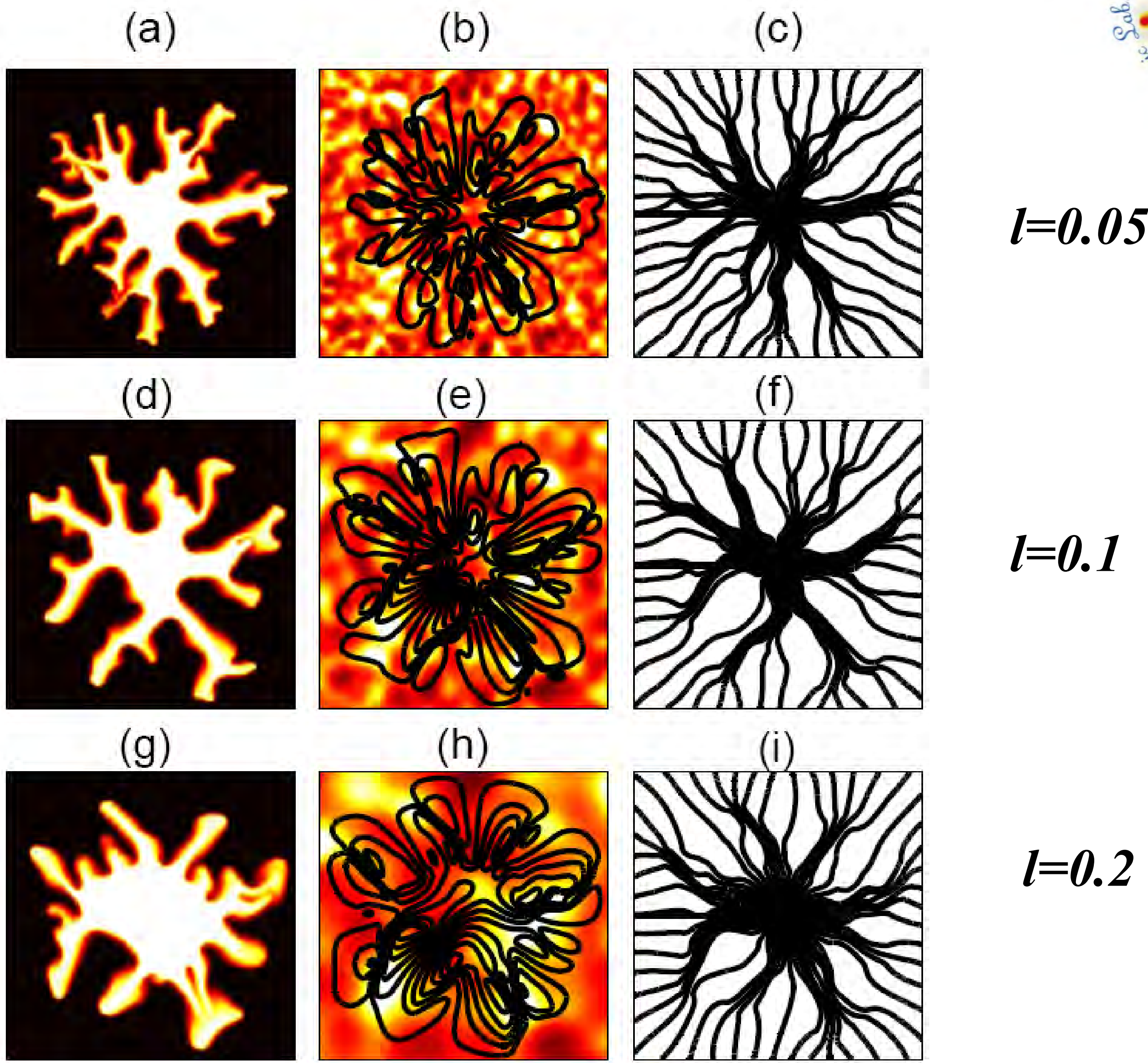
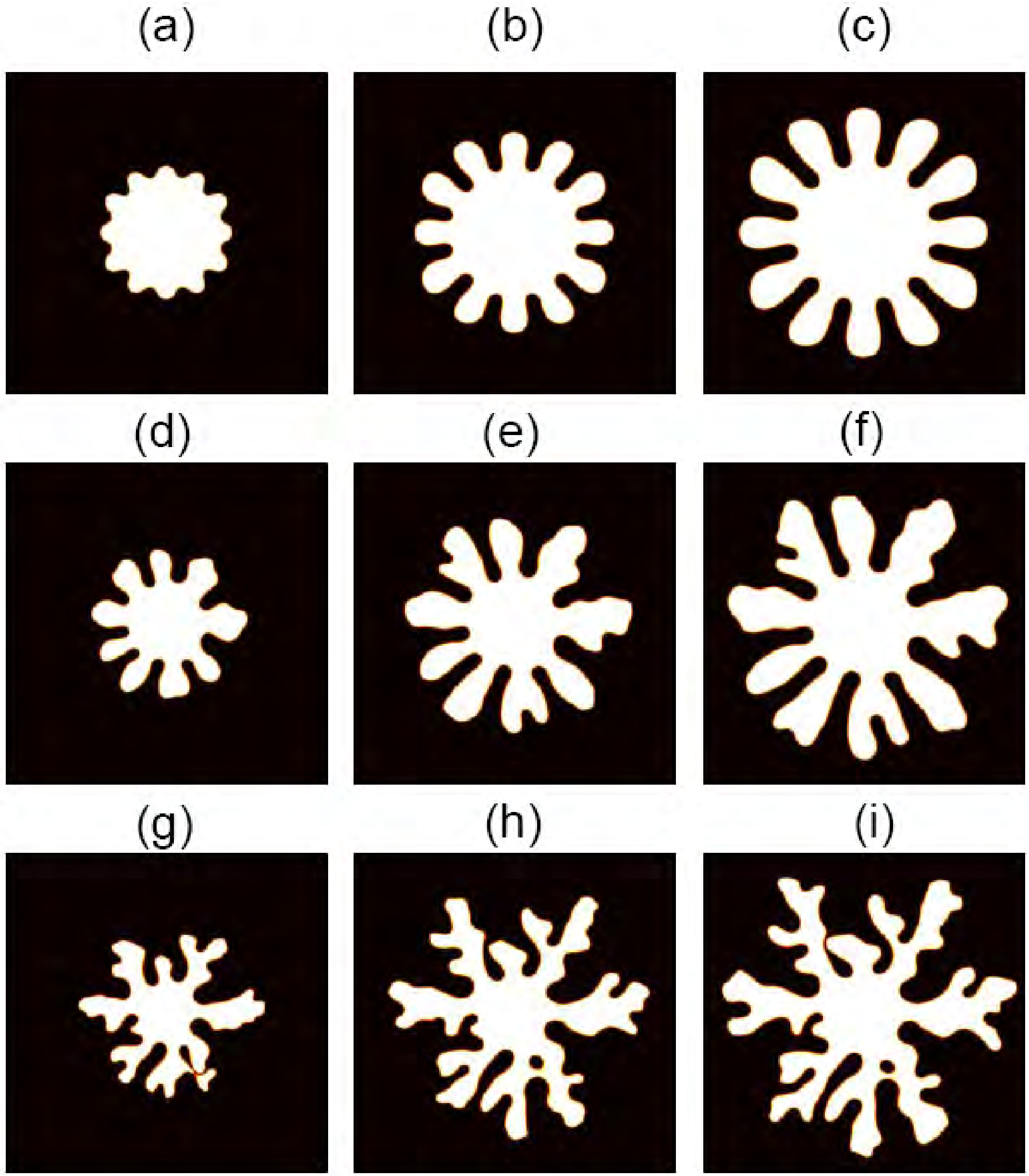
Miscible Injection : $s=0.6$



Immiscible Injection : *l=0.02*



s=0 (Homo)

s=0.3

s=0.6

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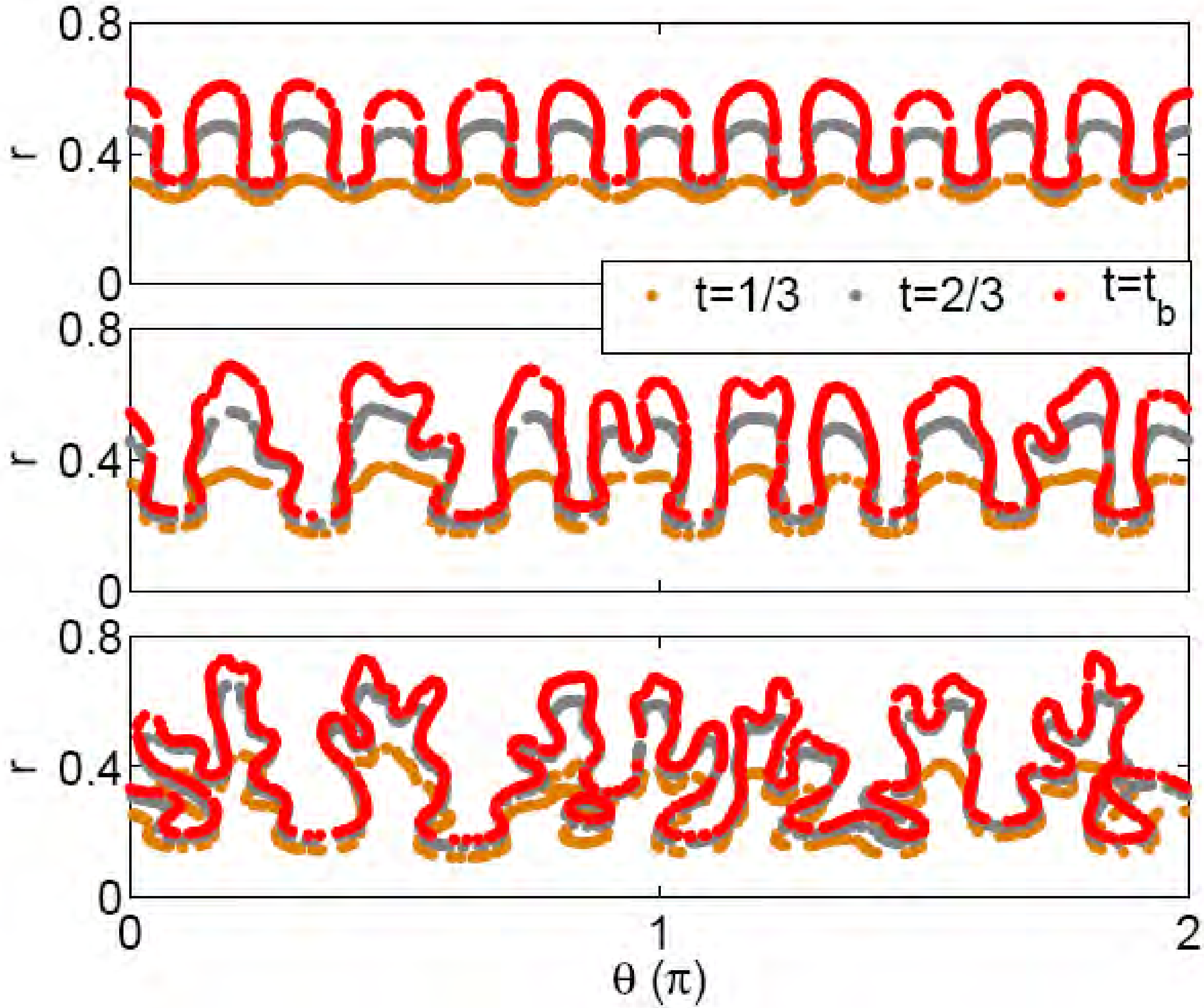
Immiscible Injection : *l=0.02*



Evolutions of fingering interfaces presented in a polar coordinate at t=1/3, 2/3 and terminated time for s=0, 0.3 and 0.6.

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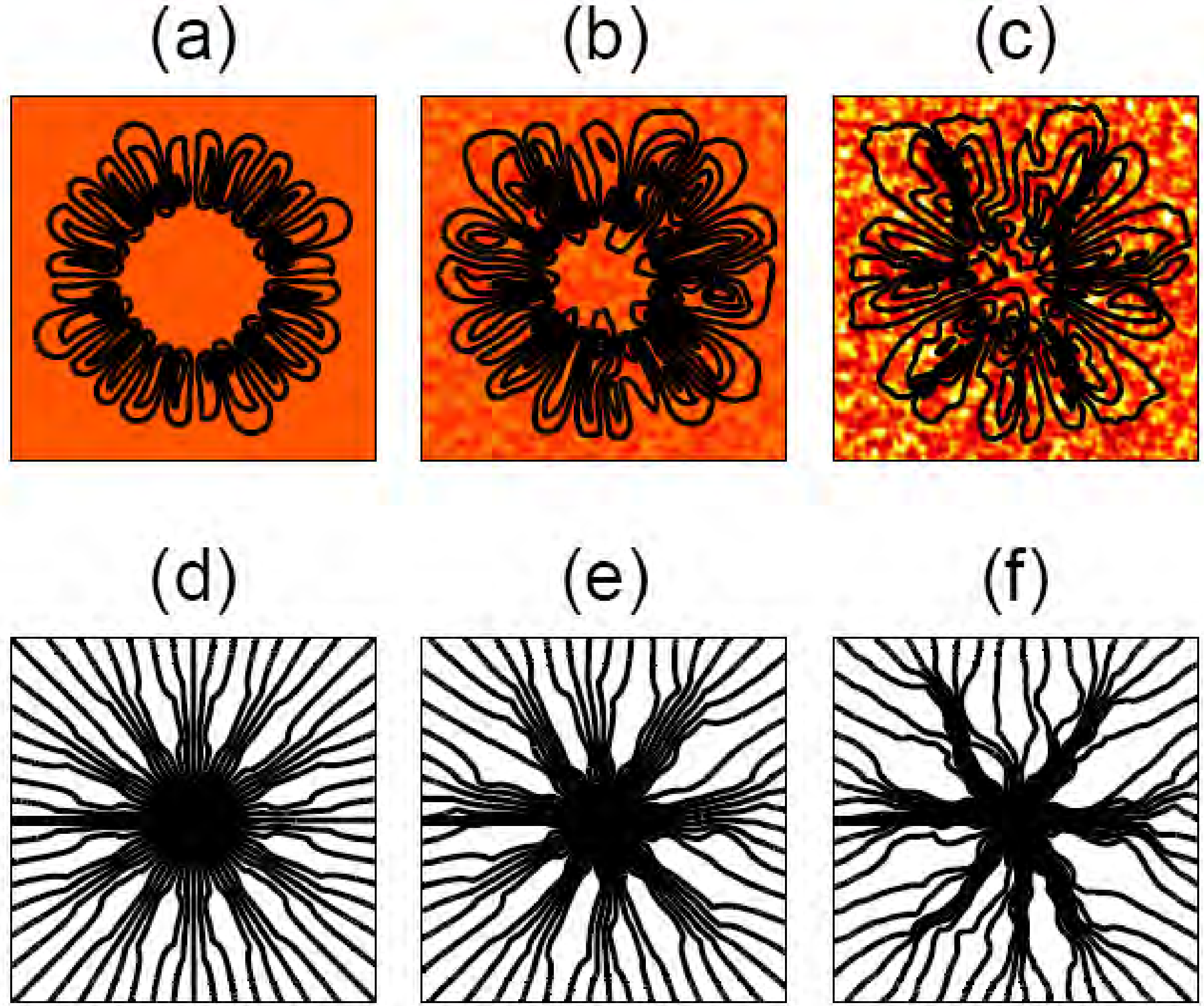
Immiscible Injection : *l=0.02*



The rotational component of streamlines superimposed on the correspondent permeability distribution (top row) and total streamlines (bottom row) for s=0, s=0.3 and s=0.6.

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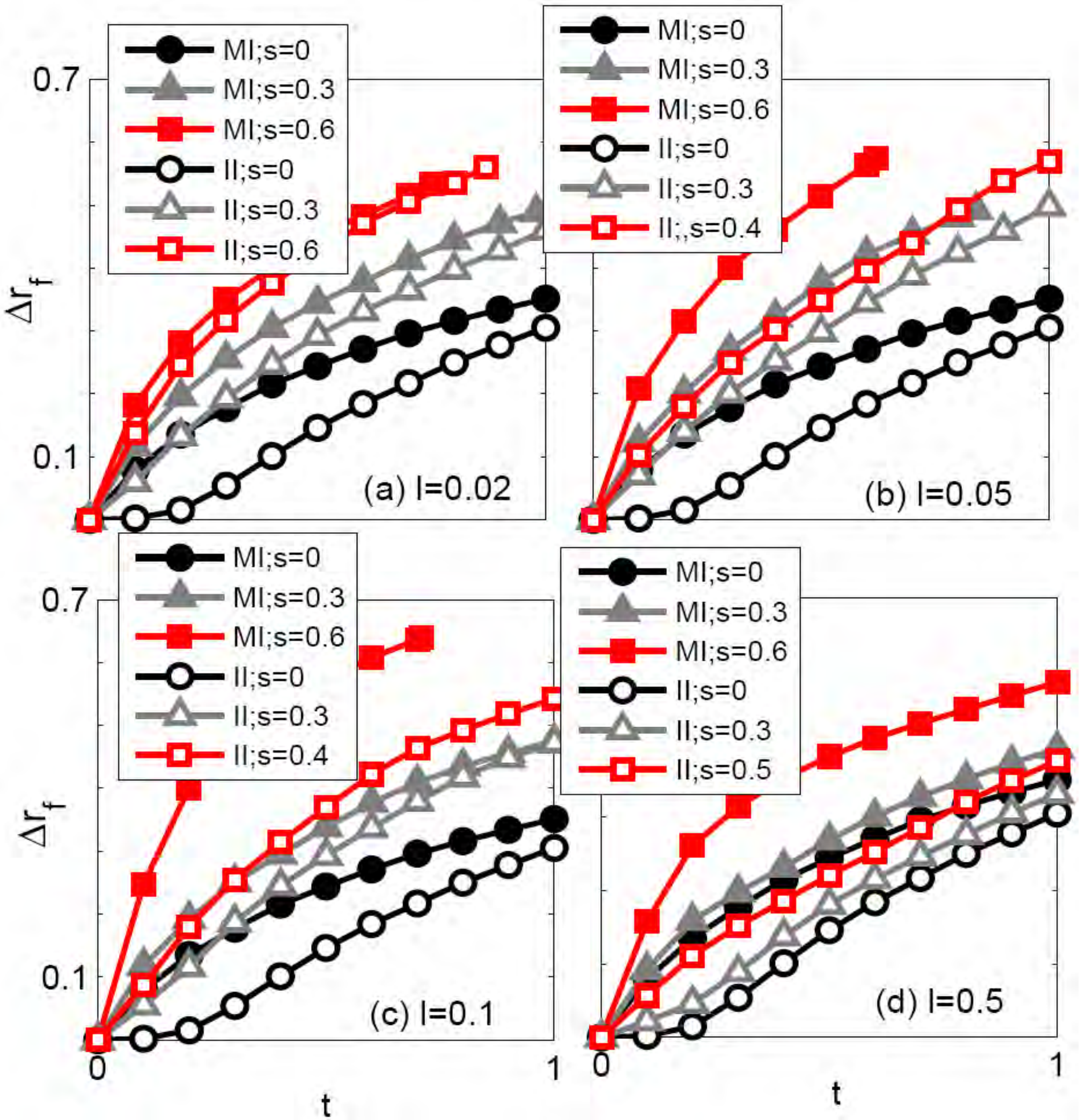
Channeling Width

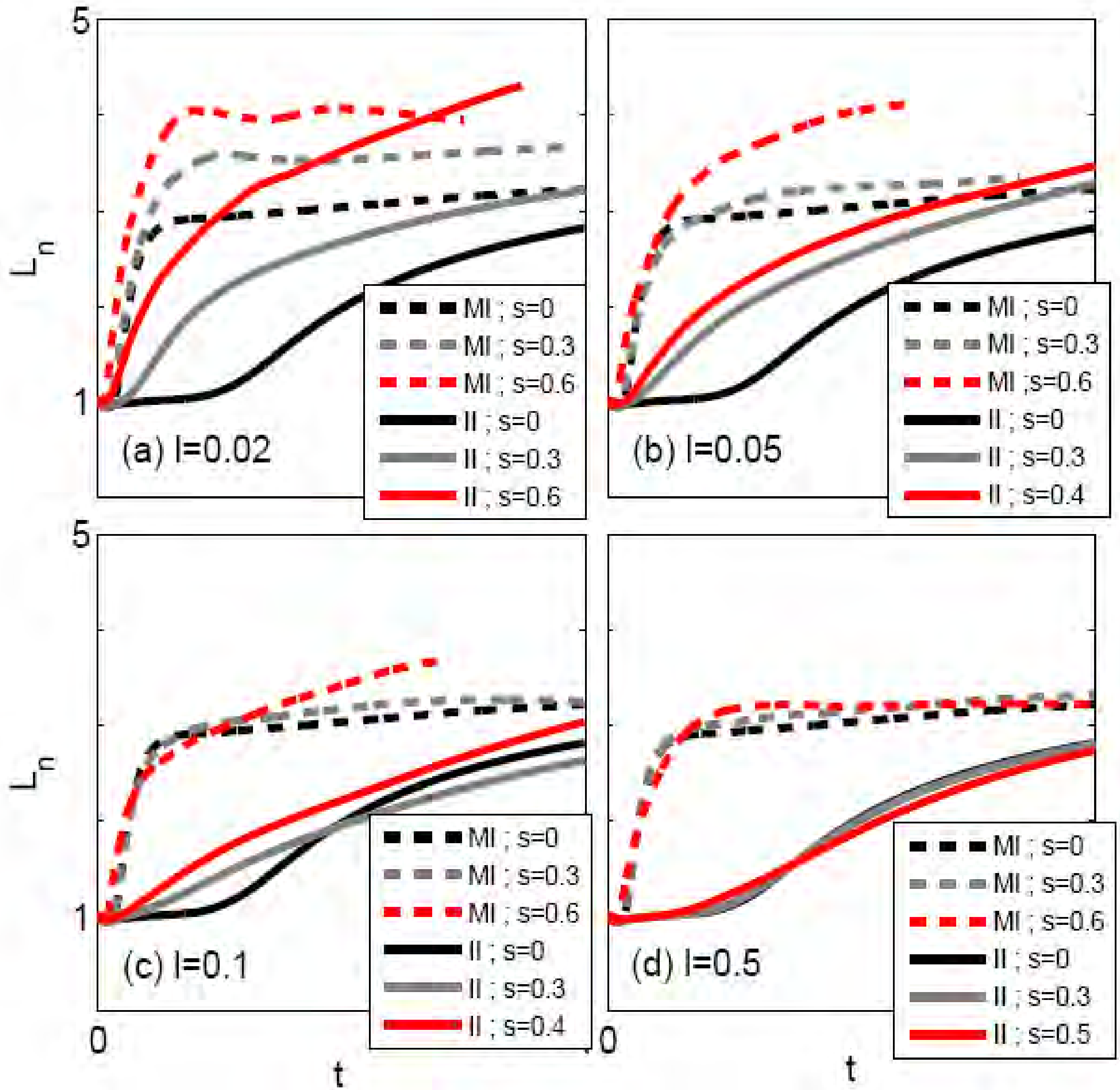


(1) Monotonically increased by larger variations

(2) Resonant effects : maximum at intermediate correlation lengths

(3) More significant in miscible condition without constraint by surface tension

Normalized Interfacial Length



(1) Monotonically increased by larger variations in smaller correlation lengths due to actively side-branches

(2) Insignificant influences by variations in larger correlation length without vigorous size-branches

(3) More significant in miscible condition without constraint by surface tension

Summary

- Phase-field approach based on HSCH model is capable to simulate both miscible and immiscible conditions by properly taking interfacial free energy functions.

- Influences of statistical parameters dominated the permeability heterogeneity, e.g. correlations length and variation, are studied systemically.

- Channeling and side-branches are enhanced by permeability heterogeneity.

- Due to insignificant side-branches in large correlation length, Interfacial lengths show independences on variation.

Thank You!